EQUAZIONI CARDINALI DELLA STATICA

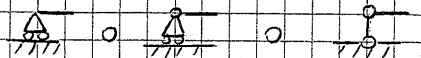
Venerdì 04/03/2016

$\begin{cases} R = 0 \\ M = 0 \end{cases}$ nello spazio: $\begin{cases} \begin{bmatrix} R_x \\ R_y \\ R_z \end{bmatrix} = 0 \\ \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = 0 \end{cases}$ nel piano: $\begin{cases} R_x = 0 \\ R_y = 0 \\ M_z = 0 \end{cases}$



$\begin{cases} R^{(a)} + R^{(r)} = 0 \\ M^{(a)} + M^{(r)} = 0 \end{cases}$ nello spazio: $\begin{cases} \begin{bmatrix} R_x^{(a)} \\ R_y^{(a)} \\ R_z^{(a)} \end{bmatrix} + \begin{bmatrix} R_x^{(r)} \\ R_y^{(r)} \\ R_z^{(r)} \end{bmatrix} = 0 \\ \begin{bmatrix} M_x^{(a)} \\ M_y^{(a)} \\ M_z^{(a)} \end{bmatrix} + \begin{bmatrix} M_x^{(r)} \\ M_y^{(r)} \\ M_z^{(r)} \end{bmatrix} = 0 \end{cases}$ nel piano: $\begin{cases} R_x^{(a)} + R_x^{(r)} = 0 \\ R_y^{(a)} + R_y^{(r)} = 0 \\ M_z^{(a)} + M_z^{(r)} = 0 \end{cases}$

Appoggio semplice o carrello



(biella)

$\begin{cases} u_x \neq 0 \\ u_y = 0 \\ \varphi \neq 0 \end{cases} \Rightarrow \begin{cases} X = 0 \\ Y \neq 0 \\ M = 0 \end{cases}$

PRESTAZIONE CINEMATICA — PRESTAZIONE STATICA



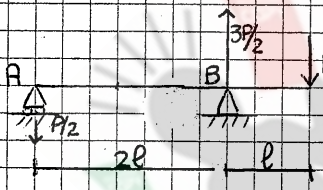
1 equazione → blocca 1 grado di libertà ⇒ MOLTEPLICITÀ = 1

Appoggio doppio o cerniera



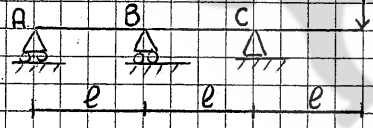
$\begin{cases} X \neq 0 \\ Y \neq 0 \\ M = 0 \end{cases}$ MOLTEPLICITÀ = 2

Esempio



pdo A $\begin{cases} X_B = 0 \\ Y_A + Y_B - P = 0 \\ Y_A \cdot 2\ell + P \cdot \ell = 0 \end{cases}$ $\begin{cases} X_B = 0 \\ Y_A = -P/2 \\ Y_B = 3P/2 \end{cases}$

Esempio



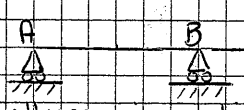
pdo C $\begin{cases} X_B = 0 \\ Y_A + Y_B + Y_C - P = 0 \\ Y_A \cdot 2\ell + Y_B \ell + P \cdot \ell = 0 \end{cases}$?

Struttura cinematicamente iperdeterminata
Sistema staticamente indeterminato

Struttura : insieme delle travi e dei vincoli

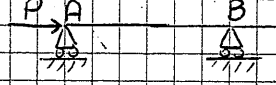
Sistema : insieme della struttura e delle forze e momenti agenti

Esempio

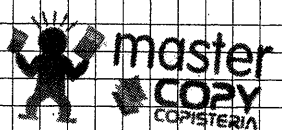


pdo B $\begin{cases} ? \quad 0 = 0 \\ Y_A + Y_B - P = 0 \\ Y_A \cdot 2\ell + P \cdot \ell = 0 \end{cases}$

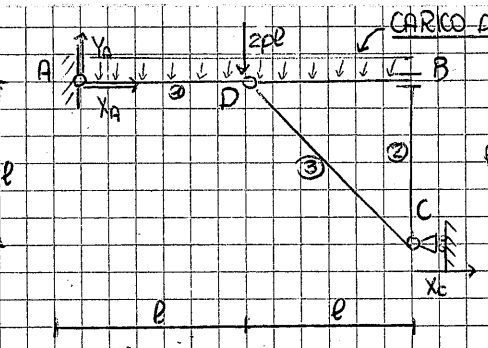
Struttura cinematicamente ipodeterminata o labile
Sistema staticamente determinato



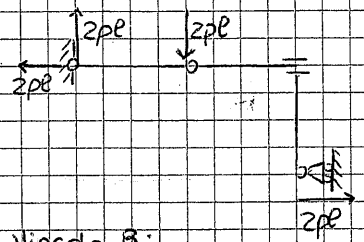
Sistema staticamente impossibile



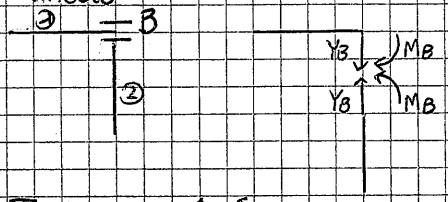
Esercizio:



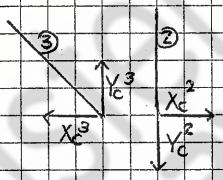
$$\begin{cases} X_A + X_C = 0 \\ Y_A - 2p\ell = 0 \\ \text{pdo B} \quad X_C \cdot \ell - 2p\ell \cdot \ell = 0 \end{cases} \qquad \begin{cases} X_A = -2p\ell \\ Y_A = 2p\ell \\ X_C = 2p\ell \end{cases}$$



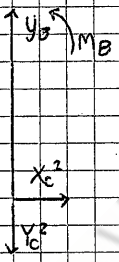
Vincolo B:



Vincolo C:



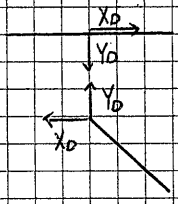
Trave 2:



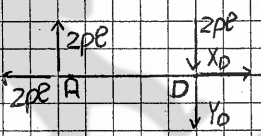
$$\begin{cases} X_C^2 = 0 \\ Y_B = Y_C^2 \\ M_B = 0 \end{cases}$$



Vincolo D:

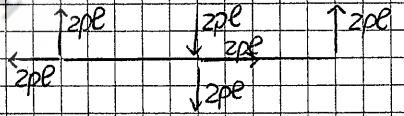


Trave 1:



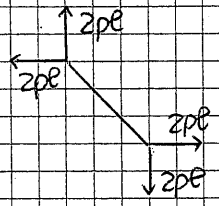
$$\begin{cases} X_D = 2p\ell \\ Y_D = Y_B \\ 2p\ell^2 + Y_B\ell = 0 \end{cases}$$

$$\begin{cases} X_D = 2p\ell \\ Y_B = -2p\ell \\ Y_D = 2p\ell \end{cases}$$

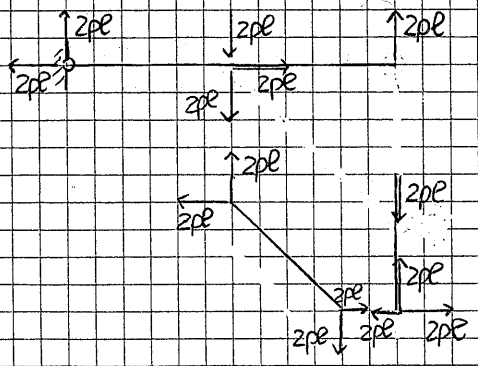


$Y_C^2 = 2p\ell$

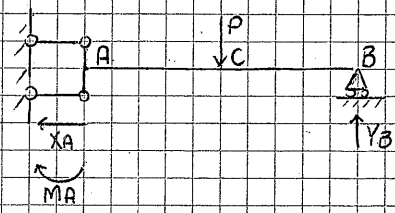
Trave 3:



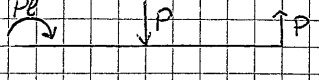
$$\begin{cases} X_C^3 = 2p\ell \\ Y_C^3 = 2p\ell \end{cases}$$



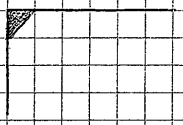
Esercizio:



$$\begin{cases} X_A = 0 \\ Y_B = P \\ M_A + P\cdot\ell = 0 \end{cases} \quad \text{pdo } B \qquad M_A = -P\ell$$



Incastro interno

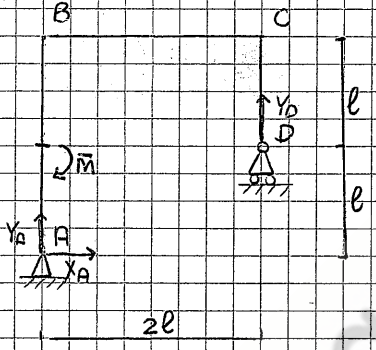


$$\begin{cases} X \neq 0 \\ Y \neq 0 \\ M \neq 0 \end{cases}$$

Possiamo considerarlo come un unico corpo

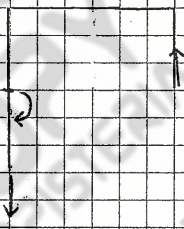
Ogni punto di una trave si può vedere come un incastro interno

Esercizio:



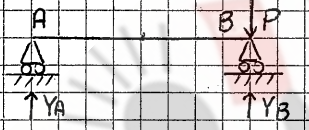
$$\begin{cases} X_A = 0 \\ Y_A = -Y_D \\ -\bar{M} + Y_D \cdot 2\ell = 0 \end{cases} \quad \text{pdo } A$$

$$y_D = \frac{\bar{M}}{2\ell} \qquad y_A = -\frac{\bar{M}}{2\ell}$$

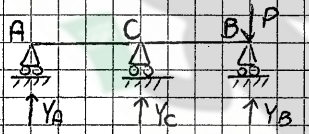


lunedì 07/03/2016

Esempio:



$$\begin{cases} 0 = 0 \\ Y_A + Y_B = P \\ Y_A \cdot \ell = 0 \end{cases} \quad \text{pdo } B \qquad \begin{matrix} Y_A = 0 \\ Y_B = P \end{matrix}$$



$$\begin{cases} 0 = 0 \\ Y_A + Y_C + Y_B = P \\ Y_A \cdot \ell + Y_C \cdot \frac{\ell}{2} = 0 \end{cases} \quad \text{pdo } B$$

Teorema di Rouchè-Capelli

Esistono soluzioni per un sistema se e solo se il rango (o caratteristica) della matrice completa è uguale a quello della matrice incompleta

$$k_0 = k_1 = k$$

Indico con n il numero di incognite:

- $n = k \Rightarrow$ il sistema ammette un'unica soluzione
- $n < k \Rightarrow$ il sistema non ammette soluzione
- $n > k \Rightarrow$ il sistema ammette ∞ soluzioni : ∞^{n-k} (cioè posso scrivere le soluzioni in funzione di $n-k$ parametri)

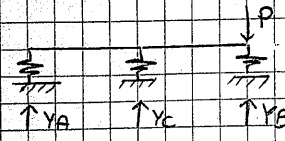
$$k_0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1/2 \end{bmatrix} = 2 \qquad k_1 = \begin{bmatrix} 1 & 1 & 1 & P \\ 1 & 0 & 1/2 & 0 \end{bmatrix} = 2$$

$$\begin{cases} Y_C = X \\ Y_A = -X/2 \\ Y_B = P - X/2 \end{cases}$$

sistema staticamente indeterminato.

Nel mondo reale i vincoli non sono perfetti e la trave non è completamente rigida: tenendo conto delle imperfezioni posso ottenere la <u>soluzione effettiva</u>, cioè quella che ottengo sperimentalmente.

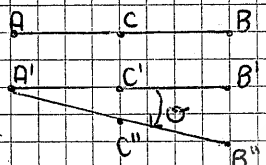
<u>Idea</u>: i vincoli sono cedevoli ⇒ posso pensare che siano molle.



La struttura si deforma: i punti di applicazione cambiano ma questo è trascurabile.

$Y_A = k \cdot V(A)$ $\quad Y_B = k \cdot V(B)$ $\quad Y_C = k \cdot V(C)$ con $V(x)$ intendo gli spostamenti verticali.

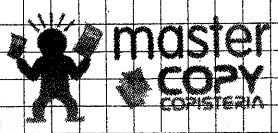
Essendo l'asta rigida:



<u>METODO DEGLI SPOSTAMENTI:</u>

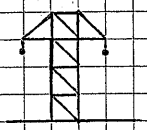
Posso esprimere $V(A)$, $V(B)$ e $V(C)$ in funzione di δ e ϑ

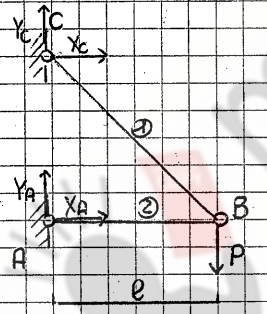
In questo modo $n=2=k$ ⇒ esiste un'unica soluzione



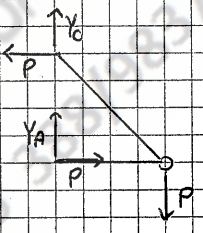
giovedì 10/03/2016

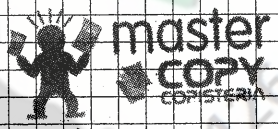
Esercizio



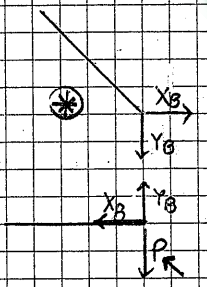


$$\begin{cases} X_A + X_C = 0 \\ Y_A + Y_C = P \\ \text{polo C} \quad P\ell - X_A \cdot \ell = 0 \end{cases} \qquad X_A = P \qquad X_C = -P$$

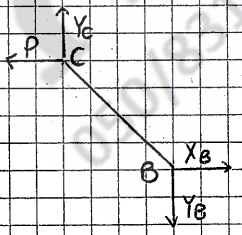




Vincolo B:

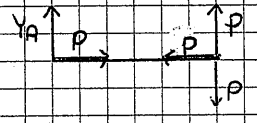


Trave 1:

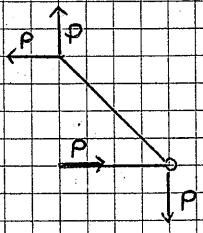


$$\begin{cases} X_B = P \\ Y_C = Y_B \\ \text{polo B} \quad P \cdot \ell = Y_C \cdot \ell \end{cases} \qquad \frac{Y_B = P}{Y_C = P}$$

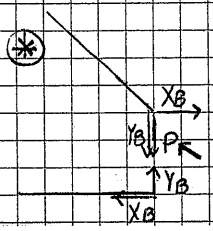
Trave 1:



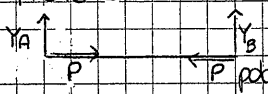
$Y_A = 0$



Vincolo B:



Trave 2:



$$\begin{cases} X_B = P \\ Y_A + Y_B = 0 \\ \text{polo A} \quad Y_B \cdot \ell = 0 \end{cases} \qquad Y_B = 0 \;,\; Y_A = 0$$

Le reazioni interne Y_B vengono diverse perchè sono studiate in punti diversi.

Lunedì 14/03/2016

Un sistema è STATICAMENTE NON DETERMINATO quando le equazioni cardinali della statica non consentono, da sole, di determinare univocamente tutte le reazioni vincolari esterne e interne

$V(C) = \delta + \frac{\ell}{2}\cdot \text{Sen}\vartheta \qquad V(B) = \delta + \ell \cdot \text{Sen}\vartheta \qquad V(A) = \delta$

IPOTESI DI "PICCOLEZZA": $|\delta| \ll \ell$ e $|\vartheta| \ll 1$

$\Rightarrow \text{Sen}\vartheta \sim \vartheta$

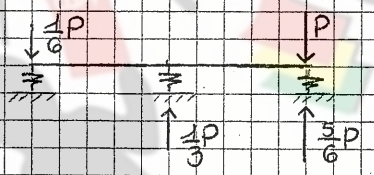
$V(C) = \delta + \frac{\ell}{2}\vartheta \qquad V(B) = \delta + \ell\vartheta \qquad V(A) = \delta$ ← linearizzo.

$\Rightarrow Y_C = k(\delta + \frac{\ell}{2}\vartheta) \qquad Y_B = k(\delta + \ell\vartheta) \qquad Y_A = k\cdot\delta$

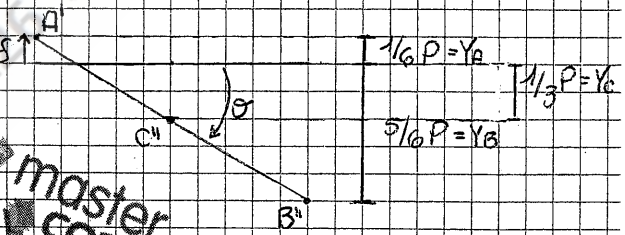
$$\begin{cases} Y_A + Y_B + Y_C = P \\ Y_A \cdot \ell + Y_C \frac{\ell}{2} = 0 \end{cases} \qquad \begin{cases} k\delta + k\delta + k\ell\vartheta + k\delta + k\frac{\ell}{2}\vartheta = P \\ k\delta\ell + k\delta\frac{\ell}{2} + k\frac{\ell^2}{4}\vartheta = 0 \end{cases}$$

$$\begin{cases} 3k\delta + \frac{3}{2}k\ell\vartheta = P \\ \frac{3}{2}k\delta\ell = -\frac{1}{4}k\ell^2\vartheta \end{cases} \qquad \begin{cases} \delta = -\frac{1}{6}\ell\vartheta \\ -\frac{1}{2}k\ell\vartheta + \frac{3}{2}k\ell\vartheta = P \end{cases} \qquad \begin{cases} \delta = -\frac{1}{6}\ell\vartheta = -\frac{1}{6}\frac{P}{k} \\ \vartheta = \frac{P}{k\ell} \end{cases}$$

$\Rightarrow Y_A = -\frac{P}{6} \qquad Y_C = k\left(-\frac{1}{6}\frac{P}{k} + \frac{\ell}{2}\frac{P}{k\ell}\right) = \frac{1}{3}P \qquad Y_B = P + \frac{1}{6}P - \frac{1}{3}P = \frac{5}{6}P$



$\delta < 0$: la trave si sposta in alto $\Rightarrow \delta\uparrow$



MODELLO LINEARE DELL'EQUILIBRIO

Condizioni imposte per risolvere il sistema:

1) ipotesi di piccolezza
2) vale la legge di Hooke - comportamento elastico lineare
3) scrivo le equazioni di equilibrio nella configurazione iniziale

$\Rightarrow$ ottengo un'unica soluzione.

Tale soluzione dipende linearmente dai dati